Algebra! What's the point?!

Algebra is a branch of mathematics that substitutes letters for numbers, and an algebraic equation represents a scale where what is done on one side of the scale is also done to the other side. We use algebraic skills in all sorts of situations, calculating time to carry out a task, profit, how much to budget for food etc.

Did you know?

The **algebra** originated from the Arabic word "al-jabr." Its history began in ancient Egypt and Babylon. The Persian mathematician Muhammed ibn Musa Al-Khwarizmi is credited as one of the forefathers of algebra.

Without a fundamental understanding of algebra, it can be argued that it is impossible to deal with geometry, statistics, and many other areas of mathematics.

Algebra originally referred to a surgical procedure, and still is used in that sense in Spanish, while the mathematical meaning was a later development.

"if b=4 then 3b is 34" This would be incorrect because…

In algebra we use letter and symbols to represent unknown quantities. We sometimes use the letter x this can be easily mistaken for the multiplication symbol. Therefore if we want to write $3 \times b$ we write $3b$. In the example above if we now say that b= 4 then $3b = 3 \times 4 = 12$.

We could also write $3b = b + b + b$ If we think about this as b *is a bag containing an unkown number of marbles*.
If I have 3 bags then I have $b + b + b$ *marbles*

PiXL© Maths
KS3 Spine
ALGEBRA

"Change the side change the sign"

When we are solving equations we are trying to find the value of the **unknown** quantity.
If we think about $x + 12 = 23$ then we can think about this as "something plus 12 equals 23" in order to calculate the "something" we need to do the calculation $23 - 12$. We haven't *changed the sign* we have **undone** or reversed the operation.
$15 = 2x + 3$ means "a number is multiplied by 2 then we add 3, the result of this is 15" to undo or reverse this we first subtract 3 from both sides- **WE NEED TO KEEP IT BALANCED**.
Giving $12 = 2x$
then we need to divide both sides by 2.
$$6 = x$$

"Use FOIL for expanding brackets"

At some point during secondary school you will be asked to expand brackets. One method used is FOIL, this stands for **First Outside Inside Last (FOIL).** But ask WHY does this work? We could just as easily use OLIF or FOLI etc.
Let's look at a number version to help see what we are doing.
$$23 \times 45$$
We can split this up into
$(20 + 3) \times (40 + 5)$
Now lets look at it on a grid

X	20	3
40	800	120
5	100	15

So we have calculated
20×40
20×5
40×3
3×5

Does it matter if we had done these calculations in a different order? Try them see if you get the same answer

Basic Rules

$$bc = cb = b \times c$$
$$5h = 5 \times h$$
$$d \times d = d^2$$
$$f \times f \times f = f^3$$
$$a(b + c) = ab + ac$$
$$g + g + g + g = 4g$$

Test Me!

Each question matches the checklist of the basic skills
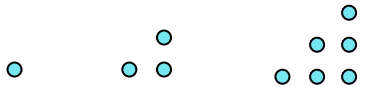Find possible values for the missing number, there is more than one solution.

1. $\boxed{} + 5 = 12 - \boxed{}$

2. $\boxed{} + 8 = 15 - \boxed{}$

3. Find the next three terms in the sequence and describe the rule.

$$5, \ 8, \ 11, \ 14, \ 17\ldots\ldots\ldots$$

4. Draw the next pattern in the sequence. Explain what is happening.

5. I am thinking of a number, I add 4 to it then divide by 2. My answer is 7. What was my starting number? Can you show this using algebra?

6. The formula for calculating the cost of a taxi is £1.20 per mile plus a £2.00 fixed charge. How much will it cost to travel;
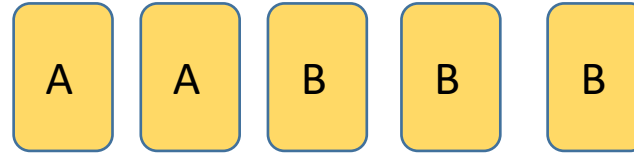a) 4 miles
b) 22 miles
c) Write the a formula to calculate the cost for any number of miles.

7. If $p = 5, \ f = 7 \ and \ g = 2$ find
a) $p + f$ b) pf c) $pf + g$

Challenge

1. You have five number cards turned over. The total of the cards is 20. What could the number be?
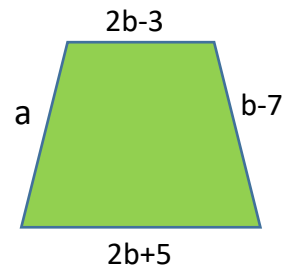
| A | A | B | B | B |

2. On a grid with axis -5 to 5 in both the x and y direction, plot all the pairs of whole numbers that add to 8. What do you notice?

3. The first 4 terms of a sequence are 6   10   14   18.
a)   Find the next three terms in the sequence
b)   Josh says that 214 is in the sequence. Is he correct? Explain your answer.

4. Write an expression for the perimeter of the shape.

(trapezium with sides: 2b-3 (top), a (left), b-7 (right), 2b+5 (bottom))

Extend

Each question looks similar **BUT** you will have to do a different calculation using the skills you have. When you are answering the questions try and think or write down what is the **same** what is **different** about each question.

(rectangle: 2f+5 (top), 7 (left side))
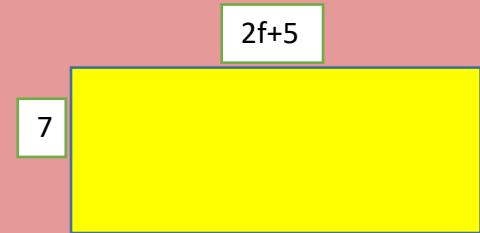
1.   Write an expression for the perimeter of the shape.
2.   If $f = 4$ what is the perimeter?
3.   Find an expression for the area of the shape.
4.   If the triangle and the rectangle have the same perimeter. Find the value of f.

(triangle with sides: $2f + 5$, $3f + 3$, $5f - 2$)

Maths KS3 **Algebra**
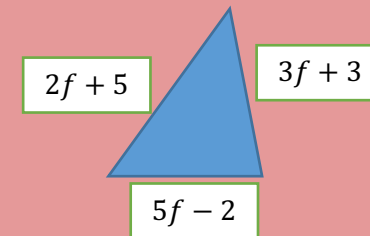
Did you know?

Without parallel lines, our buildings would be wobbly and trains would never be able to make their journey. The longest parallel line is the equator: if we take it to be a map line and don't think about changes in altitude etc., the latitude line cuts the earth in half.

Pytahagoras is perhaps a mathematician who you have heard of. He is celebrated by some as the first "real" mathematician. However, there are some who believe that the theorems named after him were actually solved by his followers. When Hippasus, a student and follower of Pythagoras, tried to find the value of $\sqrt{2}$, he realised it couldn't be written as a fraction, meaning there might be a whole new set of numbers, later known as irrational numbers. This was so challenging to the established way of thinking that Hippasus was drowned for his work!

The Egyptians needed to use a primitive form of trigonometry in order to build the pyramids. Can you imagine building the pyramids without a calculator or a computer?
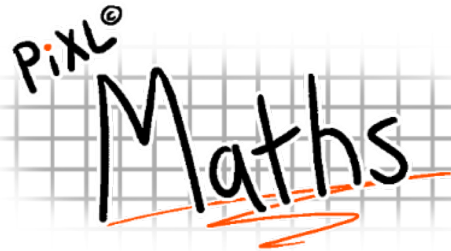
"Triangles always look the same"
Incorrect because..
*Which of these are triangles?*

ALL OF THEM! The third one is perhaps the one we are most used to seeing but be careful, the orientation doesn't change the fact it is still a triangle.

KS3 Spine
Geometry

Angle facts
You need to know which angle fact to use and how to **explain why** you have picked a particular fact. Sometimes more than one rule could be applied. Watch out though for questions that catch you out. e.g. Find the missing angle **a** and explain why.

85^0 120^0 a
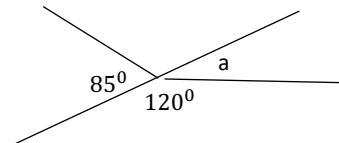
a is **NOT 85^0** , lots of people will look at this and think that a is opposite the 85^0so must be the same. The rule is about them being **vertically** opposite. We would have to use the fact about angles on a straight line here to calculate a.

"The bigger the lines the bigger the angle"
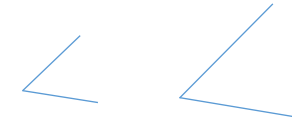
Both of these angles are the same size- Measure them using a protractor!

Angles are a measure of **turn.** The length of the line doesn't affect the magnitude of the angle.

"Parallel lines have to be the same length"

This isn't true, parallel lines remain the same distance apart but they don't have to be the same length.

Using your protractor

Do you know how to use a protractor? There are two scales. A common mistake is to use the wrong one.
BEFORE you measure an angle, look at it and estimate how big it will be. Is it **acute- less then 90^0**
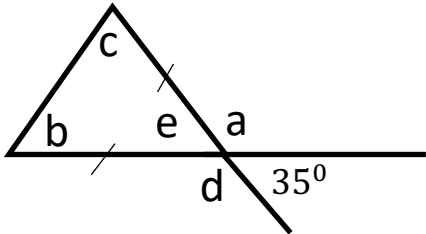**Obtuse – greater than 90^0but less than 180^0**
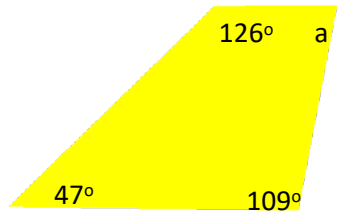**Reflex- greater than 180^0 but less than 360^0**

Test Me!

Each question matches the checklist of things you should be able to do as part of the basics for this topic.

1. Find the missing angles and explain why. If you can do this, can you find another way? Do you have to do them in alphabetical order?
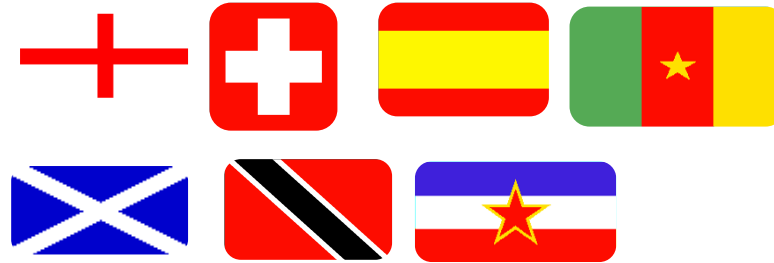

35^0

2. Find the missing angles and explain why.

126° a

47° 109°

3. Write a description of a parallelogram without drawing one.

4. On a coordinate grid, plot the following coordinates. $(1,1), (3,7), (8,5)$

5. Plot one more point on your grid to make a trapezium. Now translate this shape $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Challenge

1. Which of the flags have exactly two lines of symmetry?

2. Take a square piece of paper and fold along the diagonals. Unfold and mark any angles which are the same. Now try folding along lines of symmetry. Repeat for a rectangle. Are there the same number of equal angles in the triangles? Why? What if the starting shape was a regular pentagon? Hexagon?

3. Calculate the missing angles and give reasons.

70^0

a

d

b 42^0

c
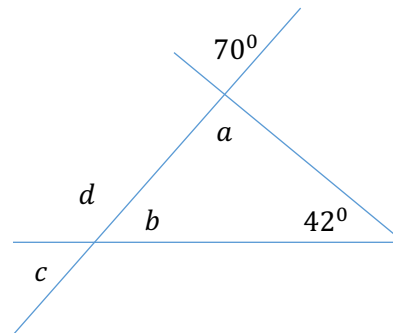
Extend
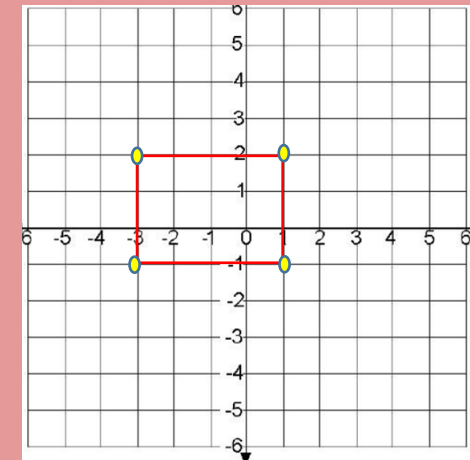
1. Translate the shape $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

2. What are the coordinates of the new vertices?

3. Which coordinates do you need to change on the original shape to make it a parallelogram?

4. Which coordinates do you need to change on the original shape to make it a rhombus?

Maths KS3 **Geometry**

Did you know?

We measure the rotation of the Earth in terms of hours. Each rotation is 24 hours in respect to the sun. The Earth's rotation is actually slowing down, so days would have been shorter in previous times. This is due to the moon's effect on the rotation.

The average depth of the ocean is about 13,000 ft. This is around 2.5 miles. Parts of the ocean will never have sunlight. This part of the ocean is called the "deep sea".

The mass of the Earth constantly changes. It gains approximately 40,000 tons each year due to particles from space.

In ancient Egypt, the Pharaoh's surveyors used measurements based on body parts to measure land and buildings.
A cubit – is measured from the elbow to the fingertip.
A digit – this is a finger, used for counting. How does this word fit with "digital age"?
The phrase "rule of thumb" comes from carpenters using a thumb to measure approximately an inch.

KS3 Spine
MEASUREMENT
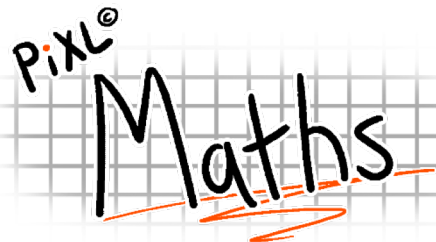
"2 hours 45 mins is the same as 2.45"
Incorrect because…
Start with an example of something you know; 2hours 30 mins is 2.5 hours *"two and a half hours"*. We wouldn't read this as *"two hours 50 minutes"*.
We need to calculate the minutes as a fraction of an hour.

$$45 \ mins = \frac{45}{60} \quad = 0.75 \ hour$$

2.3 hours in minutes would be;
$2 \times 60 = 120$
$0.3 \times 60 = 18$ Total 138 mins
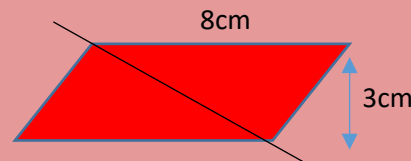
**"The area of a triangle is easy, because its just half a rectangle"**
The most common mistake is to forget to divide by two.
A triangle is half of a **parallelogram.** This is important because we use the **perpendicular height** of the triangle in the same way we do for a parallelogram.

8cm

3cm

$area \ of \ triangle$
$= \frac{1}{2}(l \times h)$
$Or \ \frac{1}{2}(Base \times height)$

"The area of a parallelogram is length times width because its just like a rectangle" incorrect because….

Rectangle

Width

Length

$$area = l \times w$$

Perpendicular height (h)

Length (l)

$$area = l \times h$$
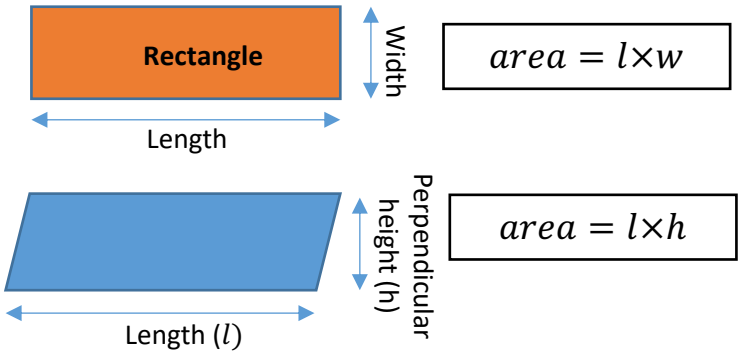
So why can't we just use the slanted edge?

If we continued to reduce the **perpendicular height** the area would reduce, but the edge would remain the same therefore it wouldn't work. e.g.

7cm

3cm

8cm

$$area = 8 \times 3$$
$$24cm^2$$
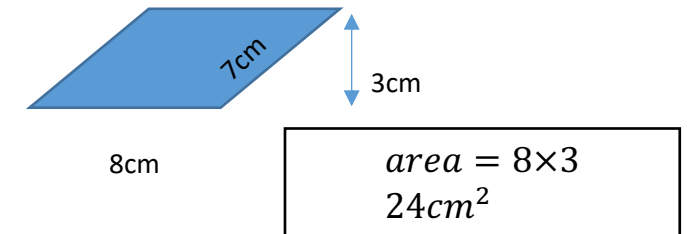
There are other ways to see this- can you see how to use triangles and rectangles to find the area? Investigate different ways to show the area of a parallelogram.

Test Me!

Each question matches the checklist of basic skill.

1. Convert the following into cm
   a) 30mm   b) 4cm 45mm   c) 12m

2. Convert the following into mm
   a) 3cm   b) 33m   c) 13cm 4mm

3. Write down the formula for the area of
   a) rectangle   b) triangle   c) parallelogram

4. Calculate the area of a triangle with base 4cm and height 12cm.

5. The two triangles are similar. Calculate the length of the missing side.
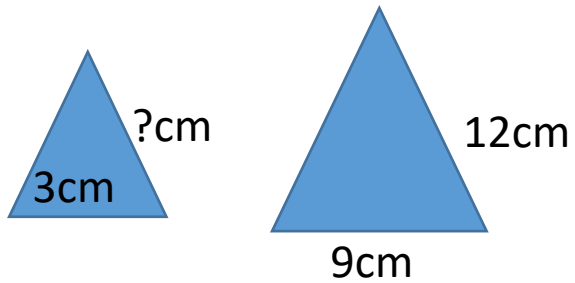
?cm

3cm

12cm

9cm

Challenge

These questions test your understanding of the misconceptions people have.

1. The time is 3.04pm a journey takes 67 minutes. What time will the journey end?

2. Convert the following into minutes
   a) 1 hour 34 mins   b) 2.65 hours

3. The area of the triangle and the parallelogram are the same. Find the perimeter of the triangle.
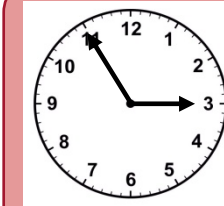
a $2a$

3

$a + 6$

3

$2a + 1$
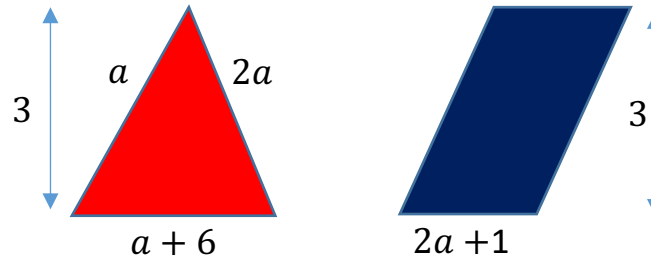
4. The area of the compound shape is $47cm^2$. Can you find possible lengths for sides a and b (whole numbers). Can you find any other possible values?(there are lots of possible answers here!)
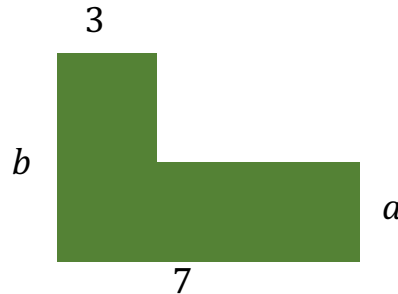
3

b

a

7

1. What time is on the clock?
2. What time will it be in 34 minutes?
3. I start watching a program at 1.20 and it finishes at the time on the clock. How many MINUTES is the programme? Write the length in hours.
4. What is the acute angle between hands?

1. Measure each side of the rectangle to the nearest mm.
2. Calculate the perimeter, round your answer to the nearest cm.
3. Draw all the lines of symmetry.

Maths KS3 Measurement

Did you know?

The word *fraction* comes from the Latin *fractus* which means "to break".

If you multiply by 1s the answer will ALWAYS be a **palindrome.** A palindrome is a number which is the same forwards as backwards.
e.g $\qquad 11111 \times 11 = 122221$

1. Start with any four digit number (that has at least two different digits).
2. Arrange the digits of the four digit number to make the largest and smallest numbers possible.
3. Subtract the smaller number from the larger one.
4. Take the answer and repeat the process.

Eventually you'll end up at 6174 or 'Kaprekar's Constant'. Interestingly, it will never take more than 7 steps to get to 6174!

Did you know? If you shuffle a pack of cards really well, there is a greater chance that this sequence of cards has NEVER been seen before in history than that it has been seen before!
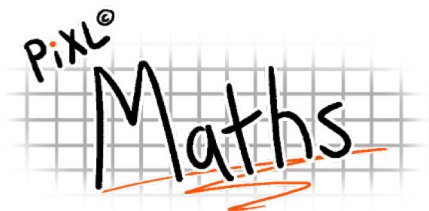
"Multiply by 10 you just add a zero"
Incorrect because….

What about decimals?
$$4.35 \times 10 = 43.5$$
We don't "*add a zero*", instead each place holder is ten times bigger.

H	T	.	$\frac{1}{10}$
	6	.	3
6	3		

Here we have 6.3×10
We can see that the **6** has now become **60** and the **0.3** has become **3**

KS3 Spine
NUMBER

"Whatever you do to the top you do to the bottom, so add the top add the bottom"

$\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$?!?! This doesn't makes sense

Think about the example above in real terms, if you have half of something and add another half you only have two quarters at the end??

When adding fractions we need a **common denominator .** In the example above we would have $\frac{2}{2}$ which is 1.

"When you multiply by a number the answer will always be bigger"

This is **sometimes true!**
$3 \times 12 = 36$ which clearly is getting bigger…
We could think about this in the context- " there are 12 eggs in a dozen and I want 3 boxes, so I want 3 of the 12 egg boxes"
What about $\frac{1}{2} \times 12$? If this was a sentence we are wanting "half of 12 which is 6"
If the temperature is -3° and it gets twice as cold we need to do $-3 \times 2 = -6$ again the answer is smaller.

"The longer the number the bigger it is"
This is sometimes true because…

What do we mean by "longer"? More digits?
3456 is certainly bigger than 23 **BUT**
1.23432 is smaller than **2** even though it has more digits.
We need to look at the PLACE VALUE of those digits to decide which is bigger.

"Two minuses makes a plus"

This is true **SOMETIMES!** $\quad -4 \times -5 = 20$
We have two minus numbers multiplied together and we get a positive answer. GREAT! … **BUT…….**
If the temperature is -5° and it **drops** by another 3 degrees it is now -8°, we did the calculation $-5 - 3 = -8$
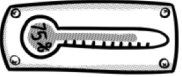We have two minus numbers but get a negative answer.
BUT… $3 - -5 = 8$
Make sure you try the questions and ask for help if you are still confused!

Test Me!

Each question matches the checklist of basic skill.

1. Write the following numbers in ascending order

3.4   3.04   3.404   3.044

2. Round the following to 1dp and 2dp
   a) 3.567   b) 0.0564   c)1.9999

3. The temperature is -5° in Berlin. The temperature in Edinburgh is 4° what is the difference in temperature?

4. Use whatever written method you like to calculate the exact solution to the following
   $a)\ 216 \div 18$ $b)\ 1728 \div 36$ $c)\ 16 \times 213$

5. If each box contains 16 books, how many books in 12 boxes?

6. Write an estimation for the following calculation
   $$1236 \div 12$$

7. Is 12 a prime number? Explain.

8. Calculate $\frac{2}{3} + \frac{1}{4}$

9. Write the decimal equivalent for $\frac{1}{2}, \frac{3}{4}, 10\%, 80\%$

10. Calculate a) 23.45×10 $b)\ 0.4563 \times 100$

11. Cancel down as far as possible

$a)\ \frac{9}{12}$ $b)\ \frac{16}{6}$ $c)\ \frac{-3}{6}$

Challenge

These questions test your understanding of the misconceptions people have.

1. Multiply the following numbers by 10 and 100
   a)0.00043   b) 23.4596

2. If I carry out the calculation 6×0.5 will my answer be bigger or smaller than 6? **Explain** why

3. Put these numbers in order, smallest first and **explain** why

4. Match the solution to the question
   -6×4 -4×-6 $-24 \div 4$ $-24 \div -6$

24   4   -6   -24

5. Jan writes down the following calculation, has she got the answer right? How do you know?

$$\frac{1}{5} + \frac{2}{3} =$$

$$\frac{3}{15} + \frac{10}{15} =$$

$$\frac{13}{30}$$

Extend

Each question looks similar **BUT** you will have to do a different calculation using the skills you have. When you are answering the questions try and think or write down what is the **same** and what is **different** about each question.

A can of lemonade costs 67p

a) If you buy one can how much change will you get from £5?
b) If you only have 33p, how much more do you need to buy 1 can?
c) How many cans can I buy with £5?
d) You buy 3 cans and pay with £5 how much change will you get?

The number 24

a) What is $\frac{2}{3} of$ 24?
b) Calculate $24 \times \frac{3}{8}$
$c)\ \frac{2}{3}$ of a number is 24, what is the original number?
d) Find $24 \div \frac{2}{3}$

Maths KS3 **NUMBER**