REASONING WITH NUMBER

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Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors.
Conjecture: a statement that might be true (based on reasoning) but is not proven.
Counterexample: a special type of example that disproves a statement.
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

This list continues and doesn't end

$3x$, $6x$, $9x$ ...

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

x x x x x x

$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2

x x / x x — $2x \times 3$

x x x / x x x — $3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

2

The first prime number
The only even prime number

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

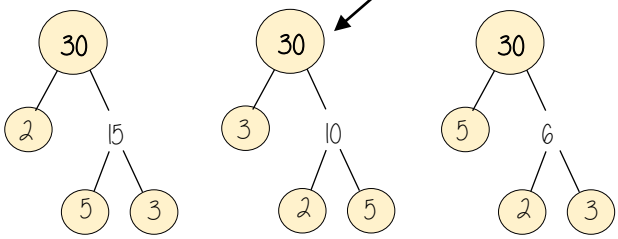
The first time their multiples match

Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$ ← Compare fractions using a LCM denominator → $\frac{6}{10}$ and $\frac{7}{10}$

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

e.g. 60 30 x 2 2 x 3 x 5 **x 2**

150 30 x 5 2 x 3 x 5 **x 5**

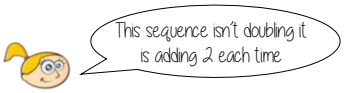
Conjectures and counterexamples

Conjecture

1, 2, 4, ...
The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples



Only one counterexample is needed to disprove a conjecture